

NEIP-94-012, McGill-94/50
 hep-ph/9411257

THE EFFECTIVE USE OF PRECISION ELECTROWEAK MEASUREMENTS

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ABSTRACT

Several reasonably model-independent formulations of the implications of new physics for precision electroweak measurements have been developed over the past years, most notably by Peskin and Takeuchi, and by Altarelli *et.al.*. These formulations work by identifying a small, but useful, set of parameters through which new physics often enters into well-measured physical observables. For the theories to which such an analysis applies, this approach greatly streamlines the confrontation with the data. Since the experimentally-allowed range for these parameters has been determined from global fits to the data, theorists need only compute their predictions for these parameters to constrain their models. We summarize these methods here, together with several recent generalizations which permit applications to wider classes of new physics, and which include the original approaches as special cases.

1. Introduction

The attainment of high precision in measurements of Z -boson properties has been perhaps the most significant experimental result in high-energy physics over the last decade. Besides testing the Standard Model (SM) to high precision, experiments at LEP and at SLC¹ are providing the first experimental winnowing of the bumper crop of theories that hope to describe the physics at energies well above 100 GeV.

*Invited talk presented to the 3rd Workshop on High-Energy Particle Physics, Madras India, January 1994.

Indeed, the data on the Z resonance is now so good that model builders ignore it at their peril. There is a very real benefit in confronting the various models of current theoretical interest with the constantly improving experimental results. One way to proceed is to simply compute the relevant observables explicitly on a model-by-model basis, and to fit the results to the data in order to constrain the model's parameter space. Unfortunately, this is a time-consuming procedure and so one is limited in the number of models which can be treated in this way. For well-motivated theories, such as the minimal supersymmetric generalizations to the standard model (MSSM) for instance, such detailed calculations may be worth the effort they require, although even here it is impractical to explore the model's entire parameter space.

Happily, there is another way to proceed which can substantially reduce the labour that is required to confront a model with the implications of the data. This alternative is based on the realization that many models often only contribute to deviations from the SM — for the well-measured observables of interest — in a limited number of ways. For instance, this could happen if all of the new particles only couple to the presently-observed ones in a restricted manner, such as through the exchange of electroweak gauge bosons. Or all of the new particles could be extremely heavy. In either case it is typically true that only a small number of independent combinations of the model's coupling constants ever appear in — and so are well constrained by — the observables that are measured. In these cases it is useful to confront the data in two steps. First, one can parameterize the well-measured observables in terms of a few independent variables which can then be constrained, once and for all, by comparing with the experiments. Next, constraints on any given model may be obtained by comparing the bounds on these parameters with the model's predictions for them as functions of its underlying couplings.

Such a two-step procedure has the advantage of separating the statistical fit to the data from model-dependent calculations. Since the data can be fit, once and for all, to a general set of parameters, it is not necessary to repeat this analysis separately for every model. Although, in principle, the permitted parameter space that is obtained for a particular theory in this way can differ somewhat from what would be obtained by a direct fit to the model, in practice the two procedures turn out to give constraints which are essentially equivalent. Furthermore, the comparative model-independence of the fit to the data in the two-step approach permits an efficient comparison of many models, and so gives a reasonably broad picture of the kinds of new physics which can produce deviations in different observables.

There are a number of similar, but not completely equivalent, ex-

amples of this type of reasoning which have become widely used in the literature.^{2,3,4,5,6} The most widely used of these are based on the parameterizations of Peskin *et.al.*,² and of Altarelli *et.al.*⁵ Both of these formalisms are very useful for describing the implications for precision electroweak measurements of a wide class of new physics. Neither of these parameterizations of the observables can encompass all models, however, and so it is important to bear in mind the limits to their applicability when attempting to use their results for comparing with any particular theory. Some well-motivated models and observables cannot be analysed completely within the framework of either approach, motivating their extension to more general situations.^{7,8} The purpose of this review is to describe those extensions which I have helped to develop, and to relate these extensions to earlier approaches. Some applications of the resulting techniques are also described.

This article is organized as follows. There are two main categories of new physics that have been considered to date: those which dominantly contribute to observables through ‘oblique’ corrections, and those for which the new physics is heavy. The next two sections are devoted to describing how each of these kinds of corrections can contribute to electroweak observables. In the next section the case of ‘oblique’ corrections is considered, while heavy, non-oblique, new physics is considered in section 3. Section 4 briefly describes a number of the applications of these techniques, and section 5 summarizes the conclusions.

2. Oblique Physics

Almost all of the observables that are presently amenable to accurate measurement can be phrased in terms of the two-particle scattering of light fermions. This is because these experiments either involve the scattering of two quarks or leptons, or the decay of an initial fermion into three lighter ones. There are three ways in which new physics can affect such experiments, given that the new particles are not themselves directly produced. It can: (a) change the propagation of the gauge bosons that can be exchanged by the fermions; (b) alter the three-point fermion – boson couplings; and (c) modify the four-point direct fermion – fermion interactions (*i.e.*: ‘box’-diagram corrections).

An important class of new-physics models contribute dominantly to precision measurements through process (a): changes to the vacuum polarizations of the electroweak bosons. Such corrections are called ‘oblique’,⁹ and when they dominate they imply *universal* modifications to light-particle scattering, in the sense that the changes depend only on the electroweak quantum numbers of the light fermion involved. Oblique corrections can be the most important when the direct couplings between the observed light fermions and any new particles are

either forbidden or highly suppressed.

2.1. General Oblique Corrections

The effects of oblique corrections on fermion scattering can be determined by examining how the gauge boson vacuum polarizations

$$\Pi_{ab}^{\mu\nu}(q) = \Pi_{ab}(q^2) \eta^{\mu\nu} + (q^\mu q^\nu \text{ terms}), \quad (1)$$

(with $a, b = \gamma, W, Z$) appear in the observables of interest.^{9,10} The contribution to these due to new physics we denote as $\delta\Pi_{ab}(q^2)$, so the full vacuum polarization is given by: $\Pi_{ab}(q^2) = \Pi_{ab}^{SM}(q^2) + \delta\Pi_{ab}(q^2)$.

In general, each of the $\delta\Pi_{ab}(q^2)$ can be arbitrary functions of q^2 , and so the vacuum polarizations in principle contain several unknown functions, each of which can potentially enter into all physical observables. Any attempt to extract general information by fitting these functions to the data might therefore seem to be doomed because of the large number of unknown quantities in comparison to the amount of data that is available. This turns out to be too pessimistic a view, however, because currently accurate measurements are performed either at very low energies, or on the Z resonance: *i.e.* only for $q^2 = m_Z^2$ and $q^2 \approx 0$. ($q^2 = m_W^2$ should also be considered to the extent that the mass and width of the W boson are thought to be sufficiently well measured.) As a result, the unknown functions, $\delta\Pi_{ab}(q^2)$, are presently only accurately sampled at these few values of four-momentum transfer.

This restriction — that precision observables only probe $q^2 \approx 0$ and $q^2 = m_Z^2$ and m_W^2 — implies that all oblique corrections to electroweak observables can be expressed in terms of six independent combinations of the various $\delta\Pi$'s.⁷ The counting proceeds as follows.

1. Inspection of the graphs with vacuum polarization insertions shows that, *a priori*, there are ten quantities to consider. For neutral-current processes at $q^2 = 0$ and $q^2 = m_Z^2$ these are: $\delta\Pi_{\gamma\gamma}(q^2)/q^2$, $\delta\Pi_{Z\gamma}(q^2)/q^2$ and $\delta\Pi_{ZZ}(q^2)$. (Notice that electromagnetic gauge invariance ensures that both $\delta\Pi_{\gamma\gamma}(q^2)/q^2$ and $\delta\Pi_{Z\gamma}(q^2)/q^2$ are well-defined at $q^2 = 0$.) $\delta\Pi'_{ZZ}(m_Z^2)$, with the prime denoting differentiation with respect to q^2 , also contributes at $q^2 = m_Z^2$. Finally, charged-current observables, or those involving physical W particles, involve $\delta\Pi_{WW}(q^2)$ at $q^2 = 0$ and m_W^2 , as well as $\delta\Pi'_{WW}(m_W^2)$.
2. Of these ten possible parameters, three combinations can never lead to observable deviations from the SM, since they can be absorbed into SM renormalizations. For instance, they can be absorbed into renormalizations for the electroweak gauge potentials, W_μ^a , B_μ , and of the Higgs vev , $\langle\phi\rangle$. This reduces the total number of possible oblique parameters to seven.

3. Finally, at the present levels of accuracy, new-physics contributions to $\delta\Pi_{\gamma\gamma}(m_Z^2)$ are not detectable. This is because this term contributes purely through photon exchange, which is not resonantly enhanced when $q^2 = m_Z^2$. As a result, the influence of $\delta\Pi_{\gamma\gamma}(m_Z^2)$ is suppressed by $O(\Gamma_Z/m_Z) \sim 0.03$ in comparison to the effects of the Z -mediated terms $\delta\Pi_{Z\gamma}(m_Z^2)$ and $\delta\Pi_{ZZ}(m_Z^2)$. We are therefore left with six measurable oblique parameters.

There is obviously a great deal of freedom in how to parameterize this six-dimensional parameter space. A convenient way to define the six oblique parameters is:

$$\begin{aligned}
\frac{\alpha S}{4s_w^2 c_w^2} &= \left[\frac{\delta\Pi_{ZZ}(m_Z^2) - \delta\Pi_{ZZ}(0)}{m_Z^2} \right] - \frac{(c_w^2 - s_w^2)}{s_w c_w} \delta\hat{\Pi}_{Z\gamma}(0) - \delta\hat{\Pi}_{\gamma\gamma}(0), \\
\alpha T &= \frac{\delta\Pi_{WW}(0)}{m_W^2} - \frac{\delta\Pi_{ZZ}(0)}{m_Z^2}, \\
\frac{\alpha U}{4s_w^2} &= \left[\frac{\delta\Pi_{WW}(m_W^2) - \delta\Pi_{WW}(0)}{m_W^2} \right] - c_w^2 \left[\frac{\delta\Pi_{ZZ}(m_Z^2) - \delta\Pi_{ZZ}(0)}{m_Z^2} \right] \\
&\quad - s_w^2 \delta\hat{\Pi}_{\gamma\gamma}(0) - 2s_w c_w \delta\hat{\Pi}_{Z\gamma}(0) \tag{2} \\
\alpha V &= \delta\Pi'_{ZZ}(m_Z^2) - \left[\frac{\delta\Pi_{ZZ}(m_Z^2) - \delta\Pi_{ZZ}(0)}{m_Z^2} \right], \\
\alpha W &= \delta\Pi'_{WW}(m_W^2) - \left[\frac{\delta\Pi_{WW}(m_W^2) - \delta\Pi_{WW}(0)}{m_W^2} \right], \\
\alpha X &= -s_w c_w \left[\delta\hat{\Pi}_{Z\gamma}(m_Z^2) - \delta\hat{\Pi}_{Z\gamma}(0) \right],
\end{aligned}$$

where s_w and c_w denote the sin and cosine of the weak mixing angle, θ_w , and α represents the electromagnetic fine-structure constant. For numerical purposes we later use $\alpha(m_Z^2) = 1/128$ and $s_w^2 = 0.23$. The quantity $\delta\hat{\Pi}_{ab}(q^2)$ denotes the ratio $\delta\Pi_{ab}(q^2)/q^2$. These definitions are chosen so that (i) the first three agree with the definitions of S , T and U that are used in Ref. 3, (ii) the remaining three quantities, V , W and X , vanish if $\delta\Pi_{ab}(q^2)$ should be simply a linear function of q^2 , and (iii) each parameter contributes to a particular kind of observable (see below).

A straightforward calculation gives expressions for the corrections to the various electroweak observables in terms of the parameters S through X . One complication arises because these corrections must be referred to the corresponding (radiatively-corrected) SM prediction. Because of the necessity for performing radiative corrections, the expressions for the SM predictions, in turn, depend on (i) which three

observables are used to infer the experimental values for the SM couplings, and (ii) unknown quantities, such as the masses, m_t , m_H , of the t quark and Higgs boson.

We follow the universal practice of using the three best-measured observables — α from low-energy electron properties, G_F from muon decay, and m_Z from LEP — as inputs for fixing the values of the SM couplings from experiment. How the parameters, S through X , appear in the final expressions depends in detail upon this choice, since new physics also affects these input observables, and so shifts the inferred values for the SM couplings. With these inputs, and with the definitions of S through X given above, U and W only enter into ‘charged-current’ quantities like the mass and width of the W boson, and of these W appears only in the W -boson width. Purely neutral-current data depend only on S , T , V and X , and of these only S and T appear in observables at low energies (for which $q^2 \approx 0$). V and X arise only in observables defined at the Z resonance. Of these two, X enters as a correction to the effective weak mixing angle, and so contributes to asymmetries such as A_{LR} or A_{FB} . V , on the other hand, drops out of these asymmetries, but instead changes the Z partial widths, since it alters the normalization of the Z -fermion couplings. The numerical expressions for these observables as functions of S through X are summarized in Table I.

The expressions from Table I can now be compared with the data to obtain bounds on the phenomenologically allowed range for the parameters S through X . In making this comparison we use the values $m_t = 150$ GeV and $m_H = 300$ GeV for computing the SM prediction. The sensitivity of our results to these assumptions is addressed in section 4. The one- σ allowed ranges for the oblique parameters which result from the fit to the data of Ref. 7 are then given by:

$$\begin{aligned} S &= -0.93 \pm 1.7; & T &= -0.67 \pm 0.92; & U &= -0.6 \pm 1.1; \\ V &= 0.47 \pm 1.0; & W &= 1.2 \pm 7.0; & X &= 0.10 \pm 0.58. \end{aligned} \quad (3)$$

Notice that since the parameter W only appears in the width of the W boson, it is the most poorly constrained.

2.2. When the New Physics is Heavy

There is an important special case for which the above oblique analysis simplifies considerably. This is when the lightest mass, M , for all of the new particles is much larger than m_Z . Since the scale over which q^2 varies appreciably in $\delta\Pi_{ab}(q^2)$ is set by M , in this case these functions may be well approximated by the first terms in their Taylor expansion in powers of q^2/M^2 . Since current measurements are restricted to the regime $q^2 \lesssim m_Z^2$, this approximation is controlled by powers of the small parameter m_Z^2/M^2 .

The leading contributions in this limit — *i.e.* those which are not suppressed by inverse powers of M^2 — are simply linear functions^{2,3,4} of q^2 :

$$\delta\Pi_{ab}(q^2) \approx A_{ab} + B_{ab} q^2. \quad (4)$$

(Higher-order terms have also been considered in the literature.¹¹) With this assumption three of the oblique parameters, V , W and X , vanish identically and so all new physics effects are described by the three parameters S , T and U . The expressions for observables in terms of these three parameters may therefore be obtained simply by setting $V = W = X = 0$ in Table I. A fit to the data,⁷ with V, W and X constrained to vanish, then gives the following one- σ allowed ranges for S , T and U :

$$S = -0.48 \pm 0.40; \quad T = -0.32 \pm 0.40; \quad U = -0.12 \pm 0.69. \quad (5)$$

Not surprisingly, the allowed range for S and T in this two-parameter fit is smaller than was permitted in the six-parameter fit whose results are given above.

Since the neutral-current data are completely controlled by the two parameters S and T , it has become conventional to display the results of fits to this data by plotting the ellipses of constant confidence interval in the two-dimensional S - T plane. Figure 1 displays the results of the two fits described above. Notice that, since m_W can depend on the parameter U as well as on S and T , m_W should only be included in such a plot if there are *a priori* reasons for believing U to be negligibly small.

2.3. Using Only the Z Resonance

In recent years the electroweak data on the Z resonance has become more accurate than are the older low-energy measurements. As a result it is now possible⁵ to usefully constrain new physics using *only* the data at $q^2 = m_Z^2$. As is clear from the earlier counting of parameters, such a restriction to only one value of four-momentum transfer permits a description of the data in terms of fewer oblique parameters than the six that were required when $q^2 \approx 0$ was also considered. This is true even if the new particles associated with the new physics are not heavy compared to m_Z .

As is easily verified by repeating the counting argument given earlier, only three parameters are required to describe the general oblique corrections in this case. A convenient choice for these three parameters is:¹²

$$S' = S + 4s_w^2 c_w^2 V + 4(c_w^2 - s_w^2) X,$$

$$\begin{aligned}
T' &= T + V, \\
U' &= U - 4s_w^2 c_w^2 V + 8s_w^2 X.
\end{aligned}
\tag{6}$$

With this choice, the parameters S' , T' and U' reduce to S , T and U in the limit where $V = W = X = 0$. This ensures that the dependence of all Z -pole observables on S' , T' and U' can be simply read off from Table I by replacing $(S, T, U, V, W, X) \rightarrow (S', T', U', 0, 0, 0)$.

As before, since the data on the Z resonance only depends on two parameters S' and T' , it is convenient to display the result of a fit of these two parameters to the data as a plot in the S' - T' plane. The result of such a fit to the Z data only¹³ is displayed in Figure 2.

It is noteworthy that the constraints on S' and T' from Figure 2 are comparable to those on S and T using the larger data set, including also the $q^2 \approx 0$ observables, that are used for Figure 1. This illustrates the high quality of the data that has been obtained over recent years at the Z resonance. The conceptual difference between Figures 1 and 2 remains crucial, however. Whereas a two-parameter, S' - T' , description of the Z -pole data relies only on the assumption that oblique corrections dominate, the extension of such a description to include also neutral-current measurements at $q^2 \approx 0$ relies on the additional assumption that all of the new physics is sufficiently massive to permit the neglect of V and X .

2.4. A Cultural Aside

Treating the Z -pole data by itself is very much in the spirit of the approach of Ref. 5. The formalism of these authors differs from that described so far in the following two ways, however.

1. These authors introduce three parameters, ϵ_1 , ϵ_2 , and ϵ_3 , which broadly correspond to the three parameters S' , T' and U' . Their definitions, however, are made directly in terms of the observables, and do not separate the new-physics contributions from those due to SM radiative corrections. As a result only the deviation, $\delta\epsilon_i = \epsilon_i - \epsilon_{SM}$, from the SM predictions — using the fiducial choices for m_t and m_H — are directly related to S' , T' and U' . The connection between the two sets of parameters is:^{5,14,15}

$$\delta\epsilon_1 = \alpha T', \quad \delta\epsilon_2 = -\frac{\alpha U'}{4s_w^2}, \quad \delta\epsilon_3 = \frac{\alpha S'}{4s_w^2}.
\tag{7}$$

2. A second important difference in the approach of Altarelli *et.al.* is to permit one non-oblique correction, parameterized by ϵ_b , to the Zbb vertex. The inclusion of this correction is motivated by the

large m_t -dependent contributions it receives from SM radiative corrections, and potentially from other kinds of new physics. We return to this type of term in the next section.

3. Nonoblique New Physics

Although many kinds of new physics dominantly produce oblique corrections to electroweak observables, this is certainly not true for all kinds. Examples include any new physics which preferentially couples to, say, the heavy generations. Another worthwhile generalization of the previous formalism is therefore to extend it to include nonoblique corrections. In order to be practical, however, such a generalization cannot be permitted to introduce too many new parameters, or else the utility of the confrontation with the data will be lost. Some criterion is necessary to limit and organize the number of independent interactions that need be considered.

A very natural way to provide the required organization is to assume that all of the new physics is much heavier than the electroweak scale, m_Z . In this case the implication of such new physics for current experiments can be parameterized in terms of a low-energy effective lagrangian¹⁶ such as would be obtained by integrating out all of the presently-undiscovered heavy particles. In this section the results of such an analysis⁸ are summarized.

The limit of large masses, M , for all hypothetical new particles allows their low-energy interactions to be organized according to dimension, with operators having a higher mass dimension being more suppressed by inverse powers of M . Simple dimension counting need not be the whole story, of course, as low-energy selection rules and symmetries can also help to determine the relative size of the various effective interactions. For a more complete discussion of the issues involved see Ref. 8.

3.1. *The Effective Interactions*

Consider, therefore, the most general effective interactions that are consistent with the particle and symmetry content that is appropriate for applications to processes having energies $\lesssim 100$ GeV. Since our intention is to study current experiments in this energy range, we take our particle content to include only those which already have been detected. This includes most of the SM particles, including precisely three left-handed neutrinos, but does *not* include the Higgs boson and the top quark, which we take to have been integrated out (if they indeed exist¹⁷). Due to the absence of these particles, the electroweak gauge group must be nonlinearly realized on the given fields, and so

in practice it can be completely ignored¹⁸ (apart from the unbroken electromagnetic subgroup) in what follows. The price for choosing this particle content is that the resulting effective lagrangian has to violate unitarity at energies at or below the TeV range.

Typically, any such lagrangian contains a great many effective interactions. Fortunately, there is a great deal of latitude in how the interactions can be written, since there is considerable freedom to redefine fields to simplify terms in the lagrangian. We choose to work with fields for which the kinetic and mass terms take their standard diagonal forms. The nonstandard interactions which arise in the most general effective lagrangian up to mass dimension five can then be written in the following way.⁸

1. Fermion Masses: The only possible effective interactions having dimension two or three are an arbitrary set of masses for the W and Z bosons and for each of the low-energy fermions. The fermion mass terms so obtained are indistinguishable from those which appear in the SM, with the exception of any neutrino masses. A neutrino mass matrix would have two effects. It would (i) give the neutrino mass eigenstates nonzero masses, and (ii) it would introduce unitary Cabibbo-Kobayashi-Maskawa (CKM) style mixing matrices, \tilde{V}_ℓ^{ij} into the various charged-current neutrino couplings. (Flavour off-diagonal neutrino kinetic terms, such as can arise when sterile neutrinos are integrated out, can also introduce off-diagonal neutral-current interactions, and can make the charged-current mixing matrices nonunitary.) Notice that, unlike most other new-physics corrections, these mixing angles need not be small, even if the neutrino masses are.

2. Electromagnetic Couplings: The total electromagnetic couplings of fermions are straightforward to write down:

$$\mathcal{L}_{\text{em}} = -e \left[\bar{f}_i \gamma^\mu Q_i f_i A_\mu + \bar{f}_i \sigma^{\mu\nu} (d_L^{ij} \gamma_L + d_R^{ij} \gamma_R) f_j F_{\mu\nu} \right], \quad (8)$$

where the indices i and j are to be summed over all possible flavours of light fermions, f_i . Q_i represents the electric charge of f_i , in units of the proton charge. γ_L and γ_R denote the usual projection matrices onto left- and right-handed spinors. Linear combinations of the effective coupling matrices, d_L^{ij} and d_R^{ij} , represent nonstandard magnetic- and electric-dipole moment interactions.

3. Charged-Current Interactions: The fermion charged-current interactions become:

$$\mathcal{L}_{\text{cc}} = -\frac{e}{\sqrt{2}s_w} \left[\bar{f}_i \gamma^\mu (h_L^{ij} \gamma_L + h_R^{ij} \gamma_R) f_j W_\mu^* \right]$$

$$+\bar{f}_i\sigma^{\mu\nu}(c_L^{ij}\gamma_L+c_R^{ij}\gamma_R)f_jW_{\mu\nu}^*]+cc, \quad (9)$$

where $W_{\mu\nu} = D_\mu W_\nu - D_\nu W_\mu$ is the W field strength using electromagnetic covariant derivatives, D_μ . For leptons,

$$\begin{aligned} h_L^{\nu_i\ell_j} &= \delta\tilde{h}_L^{\nu_i\ell_j} \\ &\quad + \tilde{V}_\ell^{ij} \left(1 - \frac{\alpha S}{4(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{2(c_w^2 - s_w^2)} + \frac{\alpha U}{8s_w^2} - \frac{c_w^2 (\Delta_e + \Delta_\mu)}{2(c_w^2 - s_w^2)} \right), \\ h_R^{\nu_i\ell_j} &= \delta\tilde{h}_R^{\nu_i\ell_j}, \end{aligned} \quad (10)$$

while for quarks:

$$\begin{aligned} h_L^{u_id_j} &= \delta\tilde{h}_L^{u_id_j} \\ &\quad + \tilde{V}_q^{ij} \left(1 - \frac{\alpha S}{4(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{2(c_w^2 - s_w^2)} + \frac{\alpha U}{8s_w^2} - \frac{c_w^2 (\Delta_e + \Delta_\mu)}{2(c_w^2 - s_w^2)} \right), \\ h_R^{u_id_j} &= \delta\tilde{h}_R^{u_id_j}. \end{aligned} \quad (11)$$

Here \tilde{V}_ℓ^{ij} and \tilde{V}_q^{ij} respectively denote the unitary CKM matrices for the left-handed charged current interactions of the leptons and quarks. The coefficients $\delta\tilde{h}_{L(R)}^{u_id_j}$ and $c_{L(R)}^{ij}$ represent a set of arbitrary nonstandard fermion- W couplings, and S , T and U are defined in terms of the vacuum polarizations as in the previous section. Finally, Δ_f (with $f = e, \mu$ or τ) denotes the following quantity: $\Delta_f \equiv \sqrt{\sum_i |h_L^{\nu_if}|^2} - 1$.

4. The W Mass: Although the SM contains gauge boson mass terms, these arise only in a particular linear combination, leading to a calculable mass relation for m_W in terms of the three inputs, m_Z , G_F and α : $m_W = m_W^{SM} \equiv m_Z c_w + (\text{radiative corrections})$. This relation is ruined by the generic effective gauge-boson mass terms, leading to the following nonstandard contribution to the W mass:⁸

$$m_W^2 = (m_W^{SM})^2 \left[1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{c_w^2 - s_w^2} + \frac{\alpha U}{4s_w^2} - \frac{s_w^2 (\Delta_e + \Delta_\mu)}{c_w^2 - s_w^2} \right]. \quad (12)$$

5. Neutral-Current Couplings: The general interactions between the light fermions and the Z boson can be written as follows:

$$\mathcal{L}_{nc} = -\frac{e}{s_w c_w} \left[\bar{f}_i \gamma^\mu (g_L^{ij} \gamma_L + g_R^{ij} \gamma_R) f_j Z_\mu + \bar{f}_i \sigma^{\mu\nu} (n_L^{ij} \gamma_L + n_R^{ij} \gamma_R) f_j Z_{\mu\nu} \right], \quad (13)$$

where

$$\begin{aligned}
(g_L^{ij})_{SM} &= (T_{3i} - Q_i s_w^2) \delta^{ij} \\
(g_R^{ij})_{SM} &= (-Q_i s_w^2) \delta^{ij} \\
g_{L(R)}^{ij} &= (g_{L,R}^{ij})_{SM} \left[1 + \frac{1}{2}(\alpha T - \Delta_e - \Delta_\mu) \right]
\end{aligned} \tag{14}$$

$$\begin{aligned}
&-Q_i \delta^{ij} \left(\frac{\alpha S}{4(c_w^2 - s_w^2)} - \frac{c_w^2 s_w^2 \alpha T}{c_w^2 - s_w^2} + \frac{c_w^2 s_w^2 (\Delta_e + \Delta_\mu)}{c_w^2 - s_w^2} \right) \\
&+ \delta \tilde{g}_{L(R)}^{ij}.
\end{aligned} \tag{15}$$

As before, Q_i represents here the electric charge of fermion f_i , and T_{3i} is its eigenvalue for the third component of weak isospin. $Z_{\mu\nu}$ denotes the abelian curl: $\partial_\mu Z_\nu - \partial_\nu Z_\mu$. The effective coupling matrices $\delta \tilde{g}_{L(R)}^{ij}$ and $n_{L(R)}^{ij}$ represent arbitrary sets of nonstandard couplings between the fermions and the Z boson.

These expressions may now be used to compute electroweak observables. Working to linear order in the effective couplings permits the neglect of all interactions which cannot interfere with the corresponding SM contribution. This eliminates a good many of the effective vertices that are listed above, including most flavour-changing interactions. (Ref. 8 gives a more general discussion which also includes the strongly bounded flavour-changing couplings.) The results for a number of low-energy observables (those for which $q^2 \approx 0$) are listed in Table II, and those for observables at the weak scale in Table III.

The parameters which appear in Tables II and III can be fit to the precision electroweak data, just as was done for the oblique parameters S through X . The results of such a fit⁸ are quoted in Tables IV, V and VI. In these tables the results of two types of fits are presented. In one of these (the ‘Individual Fit’) the parameter in question has been considered in isolation, with all of the other parameters set to zero by hand. This kind of fit is not realistic, but has often been considered in the literature. The second fit (the ‘Global Fit’) allows all of the parameters to be varied in fitting the data. Perhaps surprisingly, the resulting bounds on the various parameters are nevertheless quite good.

3.2. The Zbb Vertex

An important special case of the above analysis is to simply add a nonstandard dimension-four coupling between the Z boson and the b quark, in addition to the oblique corrections S , T and U . This corresponds to the approach of Ref. 5, with the parameters ϵ_1, ϵ_2 and ϵ_3 given as in the previous section, and ϵ_b given by:

$$\delta \epsilon_b \equiv \epsilon_b - \epsilon_b^{SM} = -2 \delta \tilde{g}_L^{bb}. \tag{16}$$

Notice that, like all of the flavour-diagonal neutral-current couplings, the reality of the lagrangian implies that $\delta\tilde{g}_L^{bb}$ must be real. In this case, the formulae of Tables II and III simplify considerably.

4. Applications

With the above results in place for the experimental limits on the various effective parameters, the next step is to compute the values of these parameters in terms of the couplings and masses of various underlying models. We therefore now turn to a brief summary of some of the applications to which the above formalism has been made.

4.1. m_t Dependence

The starting point for the comparison with experiment is the choice of a set of fiducial values, \hat{m}_t and \hat{m}_H , for the unknown masses, m_t and m_H , which appear in the SM contributions to various observables. We have chosen $\hat{m}_t \equiv 150$ GeV and $\hat{m}_H \equiv 300$ GeV in performing the fits, but it is natural to wonder how the SM predictions change as these masses vary.

Perhaps the simplest, and most useful, application of the techniques described in earlier sections is to approximately determine this m_t and m_H dependence. To do so, imagine both the top quark and the Higgs boson to be much heavier than m_Z . In this case, all of the implications for lower-energy observables due to these heavy particles can be phrased in terms of the effective lagrangian obtained by integrating them out of the SM. It can therefore be mimicked by choosing an appropriate m_t and m_H dependence for some of the effective couplings of section 3.

Evaluating the one-loop vacuum polarization graphs^{2, 3, 4, 5} containing top and Higgs loops gives the following large-mass dependence for the oblique parameters:

$$\begin{aligned}\delta S_{SM} &\simeq -\frac{1}{3\pi} \ln\left(\frac{m_t}{\hat{m}_t}\right) + \frac{1}{6\pi} \ln\left(\frac{m_H}{\hat{m}_H}\right) \\ \delta T_{SM} &\simeq \frac{3}{16\pi s_w^2 c_w^2} \left(\frac{m_t^2 - \hat{m}_t^2}{m_Z^2}\right) + \frac{3}{8\pi c_w^2} \left[\ln\left(\frac{m_t}{\hat{m}_t}\right) - \ln\left(\frac{m_H}{\hat{m}_H}\right)\right] \\ \delta U_{SM} &\simeq -\frac{1}{\pi} \ln\left(\frac{m_t}{\hat{m}_t}\right).\end{aligned}\tag{17}$$

Similarly, the Zbb vertex-correction graph involving a virtual top quark gives^{5, 19}

$$(\delta\tilde{g}_L^{bb})_{SM} \simeq \frac{\alpha}{8\pi s_w^2 c_w^2} \left(\frac{m_t^2 - \hat{m}_t^2}{m_Z^2}\right).\tag{18}$$

These are the *only* virtual contributions which can both (i) depend strongly, at one loop, on m_t , and (ii) contribute appreciably to electroweak observables.

4.2. Exotic Fermions

One application to which the above formalism has been applied⁸ is the determination of the constraints on the masses and couplings that are possible for hypothetical exotic fermions which can mix with the ordinary ones. For the present purposes we take ‘exotic’ to mean new fermions which transform under the $SU_L(2)$ gauge group as either left-handed singlets and/or right-handed doublets. Because of their mixing with ordinary fermions, exotic fermions of this sort can change the couplings of the ordinary fermions to the W and Z . Constraints on these mixings can be inferred from present precision measurements of these couplings.

One motivation for doing this analysis, is that it also has been performed²⁰ by fitting these models directly to the data. By comparing the bounds obtained here with those of the direct fit to the model we can learn how closely they agree with one another. One would expect the present approach to give bounds that are marginally weaker than those of a direct fit to the model, since there are typically more parameters in the general effective lagrangian than there are couplings in a particular underlying theory. In this sense our results can be considered the most conservative bounds possible. The interesting point is that the limits we obtain are not very much weaker at all, and so very little information is lost by using the simpler effective-lagrangian fit. For simplicity of presentation we restrict ourselves here to charged quarks, although neutrino mixing can be handled in much the same way.⁸

In general, mixing between the known fermions and exotic ones induces flavour-changing processes into the neutral current interactions of ordinary fermions (FCNC’s). For simplicity we ignore these types of induced couplings here, although they can be treated in a similar manner. FCNC’s are avoided if every known electroweak multiplet of fermions mixes separately with its own exotic partner, in which case the mixing can be characterized in terms of a mixing angle, $\theta_{L(R)}^i$, $i = e, \mu, \tau, u, d, s, \dots$

To make contact with our general formalism, simply integrate out all of the undiscovered exotic particles to produce the low-energy effective theory. Sufficient accuracy is obtained by working at tree level, and so it is easy to integrate out the heavy fermions: one transforms to a basis of mass eigenstates, and sets all heavy fields equal to zero. As a result we find $S=T=U=0$, and the nonstandard neutral current and

charged-current couplings of the ordinary fermions become modified by⁸:

$$\delta\tilde{g}_L^{ii} = -T_{3i} (s_L^i)^2, \quad \delta\tilde{g}_R^{ii} = +T_{3i} (s_R^i)^2, \quad (19)$$

and

$$\delta\tilde{h}_L^{u_i d_j} = -\frac{1}{2} \tilde{V}_q^{ij} [(s_L^{u_i})^2 + (s_L^{d_j})^2], \quad \delta\tilde{h}_R^{u_i d_j} = s_R^{u_i} s_R^{d_j} \tilde{U}_R^{ij}, \quad (20)$$

in which $(s_{L(R)}^i)^2 \equiv 1 - (c_{L(R)}^i)^2 \equiv \sin^2 \theta_{L(R)}^i$, and \tilde{U}_R^{ij} is a unitary CKM-type matrix for the right-handed charged-current couplings. Similar expressions also hold for leptons.⁸

It is now a simple matter to bound the mixing angles using these expressions together with the constraints of Tables V and VI. We find the following limits at 90% c.l. (defined as 1.64σ):

$$\begin{aligned} \Delta_{e,\mu} : \quad & (s_L^e)^2, (s_L^{\nu_e})^2 < 0.016; \quad (s_L^\mu)^2, (s_L^{\nu_\mu})^2 < 0.012 \\ \delta\tilde{h}_L^{ud} : \quad & (s_L^u)^2, (s_L^d)^2 < 0.02 \\ \delta g_{L,R}^{ii} : \quad & (s_R^e)^2 < 0.01; \quad (s_R^\mu)^2 < 0.09; \quad (s_R^u)^2 < 0.03; \\ & (s_R^d)^2 < 0.05; \quad (s_L^s)^2 < 0.05; \quad (s_L^b)^2 < 0.03, \end{aligned} \quad (21)$$

A comparison of the above numbers with those in the literature²⁰ confirms that they are very similar to, but marginally weaker than, those found by fitting directly to the mixing angles themselves.

4.3. Light Exotic Particles

Another application that has been examined with this formalism^{12,14,21} is the case of new exotic particles whose quantum numbers preclude their mixing with ordinary fermions.²² Such particles arise in a great many types of theories for new physics, including supersymmetric models, theories with additional generations, and technicolour models. In these examples the new particles need *not* be much heavier than the weak scale, and so we do not make this assumption here.

Unlike for the previous example, in this case the absence of tree-level mixing implies that the dominant effects of the new physics arise through its one-loop contributions to well-measured observables. This is in most cases dominated by the contributions to gauge boson vacuum polarizations, and so we consider here only oblique corrections.

Formulae for the one-loop contributions to the six oblique parameters, $S - X$, by scalars and fermions in general representations of the electroweak gauge group have been given in the literature.^{7,23,24} For

example, the one-loop vacuum polarization due to a fermion with left- and right-handed coupling constants, $k_{L(R)}^a$, to gauge boson ‘ a ’ (with $a = \gamma, W, Z$) is:

$$\delta\Pi_{ab}(q^2) = \frac{1}{2\pi^2} \sum_{ij} \int_0^1 dx f_{ab}(q^2, x) \ln \left[\frac{m_{ij}^2(x) - q^2 x(1-x)}{\mu^2} \right]. \quad (22)$$

Here $m_{ij}^2(x) \equiv m_i^2(1-x) + m_j^2x$, where m_i and m_j are the masses of the two fermions which appear within the loop, and

$$\begin{aligned} f_{ab}(q^2, x) = & \frac{k_L^a k_L^{b*} + k_R^a k_R^{b*}}{2} \left[x(1-x) q^2 - \frac{m_{ij}^2(x)}{2} \right] \\ & + \frac{k_L^a k_R^{b*} + k_R^a k_L^{b*}}{2} \left(\frac{m_i m_j}{2} \right). \end{aligned} \quad (23)$$

For example, a standard-model doublet would have $k_L^\gamma = k_R^\gamma = eQ_i$, $k_L^Z = (e/s_w c_w)[T_{3i} - Q_i s_w^2]$, $k_R^Z = (e/s_w c_w)[-Q_i s_w^2]$, $k_L^W = e/\sqrt{2}s_w$ and $k_R^W = 0$. μ^2 denotes the renormalization point, where we have renormalized using dimensional regularization and $\overline{\text{MS}}$.

Expressions such as these permit a survey of the couplings and masses that are permitted for exotic particles transforming in a variety of electroweak gauge representations.¹² For the purposes of illustration, Figure 3 displays the range of values for S' and T' that are produced by an additional generation of quarks and leptons. Notice that this region does *not* necessarily include the origin ($S' = T' = 0$) in the limit that the members of this additional generation become very heavy. This is because heavy fermions like these need not decouple from S' and T' in this limit. This makes the oblique parameters especially sensitive probes for these types of new particles.

4.4. *Is Technicolour Dead?*

The above observation, that some heavy particles need not decouple from oblique parameters like S and T as their masses get very large, has been used to argue that technicolour-like models for dynamical electroweak symmetry breaking are ruled out by the current electroweak data.^{2,3,4,5} Various estimates for the oblique parameters in these theories predict $S' \gtrsim O(+1)$, and the two-parameter $S-T$ fit to the Z -pole data excludes these values at roughly the three- σ level. (A similar conclusion has been reached by considering the anomalous Zbb coupling as well.²⁵)

This conclusion would be inescapable provided that these parameters could be reliably computed to be $O(+1)$ in technicolour models.

Unfortunately, since these are strongly coupled theories, the robustness of the various estimates is not clear.²⁶ For example, a potential loophole may exist when light particles are present in the technispectrum — such as often happens in these theories due to the appearance of pseudogoldstone bosons. In this case the analysis requires the additional parameters V through X , and the contributions of the light particles *can* be made to contribute negatively to S' and T' ,^{24,12} for some choices for their masses and mixings.

These loopholes are difficult to completely close due to the difficulty in computing with these theories. In the meantime, it must be conceded that the case against these theories remains persuasive, albeit circumstantial.

4.5. TGV's

As a final application we consider how the oblique analysis of section 2 can be used to bound²⁷ the potential existence of nonstandard self-couplings for the electroweak gauge bosons (TGV's).²⁸ Somewhat surprisingly, the full six-parameter set of oblique corrections turns out to be required for this analysis, even though all heavy particles are assumed to be much more massive than the weak scale. The need for all six parameters follows because loop-induced effects to oblique parameters are at most of order $\alpha/4\pi$, and this is the same order of magnitude as is m_Z^2/M^2 . As a result, TGV loop-induced corrections to the STU parameters can be the same size as the other quantities, VWX , and so these must be properly included.

The starting assumption is that the lightest mass, M , of any undiscovered particle is high in comparison with the weak scale, m_Z . Integrating out this heavy physics gives an effective lagrangian, defined at the scale M , which we assume to include the five CP-conserving TGV's, whose coefficients are, in a standard²⁸ notation: $\Delta\kappa_\gamma$, $\Delta\kappa_Z$, Δg_{1Z} , λ_Z , and λ_γ . The goal is to constrain the coefficients of these effective interactions using precision electroweak measurements at lower energies, $E \lesssim m_Z$.

In order to do so we imagine running the effective lagrangian defined at the scale M , down to the scale m_Z where current measurements are made. Since anomalous TGV's are not directly probed at these energies, the bounds come from the other effective interactions which are generated by the TGV's during the running down from M to m_Z . We therefore compute all of the effective interactions at the weak scale that get generated (at one loop) by loops containing TGV's.

There are two kinds of effective interactions that are produced by TGV's in this way: (i) oblique parameters, and (ii) nonstandard fermion/gauge-boson vertices. The fermion/gauge-boson vertices them-

selves come in two types: (a) those which are universal, in that they are independent of the masses of the SM fermions, and (b) those which depend strongly on the top-quark mass, m_t . Interestingly, we find that all of the vertex corrections of type (a) that are generated by TGV's can be rewritten as oblique corrections by performing a suitable field redefinition.²⁷ The entire analysis can therefore be done using the oblique parameters, together with a small number of m_t -dependent terms.

The TGV-generated oblique parameters that are produced by this analysis are displayed in Table VII, where the numerical values $\alpha(m_Z^2) = 1/128$ and $s_w^2 = 0.23$ are used. We also take $M = 1$ TeV and the effective couplings are evaluated at the low-energy scale $\mu = 100$ GeV. The quantities \hat{V} and \hat{X} incorporate the nonuniversal, m_t -dependent, contributions, and are to be used instead of V and X in the expressions for any observables which are based on the process $Z \rightarrow b\bar{b}$.

The expressions from Table VII can then be used in Table I, for the electroweak observables in terms of the oblique parameters. When all five TGV coefficients are left free in the resulting fit to the data, no useful bound is obtained, beyond the ever-present ones like perturbative unitarity. This shows that the data is not yet sufficiently accurate to constrain these quantities. A common procedure in the literature is to instead bound each TGV separately, with all of the others constrained to vanish. This type of analysis gives, in our case, the following one- σ allowed ranges²⁷:

$$\begin{aligned} \Delta g_{1Z} &= -0.033 \pm 0.031; \\ \Delta \kappa_\gamma &= 0.056 \pm 0.056; \quad \Delta \kappa_Z = -0.0019 \pm 0.044; \\ \lambda_\gamma &= -0.036 \pm 0.034; \quad \lambda_Z = 0.049 \pm 0.045. \end{aligned} \quad (24)$$

Although the bounds obtained in this way are more restrictive, they are also not realistic for any underlying theory, for which all couplings would be expected to be generated together.

5. Conclusions

This article reviews two approaches to parameterizing the effects of new physics for precision electroweak measurements. The two approaches are based on one of the following two assumptions. Either: (i) the new physics is assumed to be very heavy in comparison with the weak scale, m_Z , or: (ii) it is *not* assumed to be heavy, but it is assumed to dominantly contribute to observables through oblique corrections. These two approaches contain the popular formalisms of Peskin *et.al.*, and of Altarelli *et.al.*, as important special cases.

The advantage of parameterizing the data in this way is the efficiency with which it permits the comparison of specific models to the

data. Rather than having to perform a detailed fit to the data of every proposed theory, it is possible to fit the data once and for all to the proposed parameters. To constrain any particular model it is then simply necessary to compute these parameters in terms of the couplings of the underlying theory. A conservative estimate of the allowed range for these couplings can be found by simply using the appropriate confidence intervals for the basic parameterization. The bounds that are obtained in this way turn out to be remarkably similar to those which are obtained from model-by-model fits.

A number of phenomenological analyses have been performed using this procedure, some of which have been briefly summarized here. These calculations illustrate the potential applications of the method, and the simplicity with which it may be carried out.

6. Acknowledgements

I would like to thank the organizers of the workshop for their warm hospitality, and for their kind invitation to speak. The research described here was the result of fabulous collaborations with Peter Bamert, Steve Godfrey, Heinz König, David London and Ivan Maksymyk, and has been supported by the Swiss National Foundation, N.S.E.R.C. of Canada and les Fonds F.C.A.R. du Québec.

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$$\begin{aligned}
\Gamma_Z &= (\Gamma_Z)_{SM} - 0.00961S + 0.0263T + 0.0194V - 0.0207X \text{ (GeV)} \\
\Gamma_{b\bar{b}} &= (\Gamma_{b\bar{b}})_{SM} - 0.00171S + 0.00416T + 0.00295V - 0.00369X \text{ (GeV)} \\
\Gamma_{l+l-} &= (\Gamma_{l+l-})_{SM} - 0.000192S + 0.000790T + 0.000653V - 0.000416X \text{ (GeV)} \\
\Gamma_{had} &= (\Gamma_{had})_{SM} - 0.00901S + 0.0200T + 0.0136V - 0.0195X \text{ (GeV)} \\
A_{FB}(\mu) &= (A_{FB}(\mu))_{SM} - 0.00677S + 0.00479T - 0.0146X \\
A_{pol}(\tau) &= (A_{pol}(\tau))_{SM} - 0.0284S + 0.0201T - 0.0613X \\
A_e(P_\tau) &= (A_e(P_\tau))_{SM} - 0.0284S + 0.0201T - 0.0613X \\
A_{FB}(b) &= (A_{FB}(b))_{SM} - 0.0188S + 0.0131T - 0.0406X \\
A_{FB}(c) &= (A_{FB}(c))_{SM} - 0.0147S + 0.0104T - 0.03175X \\
A_{LR} &= (A_{LR})_{SM} - 0.0284S + 0.0201T - 0.0613X \\
m_W^2 &= (m_W^2)_{SM}(1 - 0.00723S + 0.0111T + 0.00849U) \\
\Gamma_W &= (\Gamma_W)_{SM}(1 - 0.00723S + 0.0111T + 0.00849U + 0.00781W) \\
g_L^2 &= (g_L^2)_{SM} - 0.00269S + 0.00663T \\
g_R^2 &= (g_R^2)_{SM} + 0.000937S - 0.000192T \\
g_V^e(\nu e \rightarrow \nu e) &= (g_V^e)_{SM} + 0.00723S - 0.00541T \\
g_A^e(\nu e \rightarrow \nu e) &= (g_A^e)_{SM} - 0.00395T \\
Q_W(^{133}_{55}Cs) &= Q_W(Cs)_{SM} - 0.795S - 0.0116T
\end{aligned}$$

TABLE I: Oblique Contributions to Observables

The dependence of some electroweak observables on S, T, U, V, W and X . The numerical values $\alpha(m_Z^2) = 1/128$ and $s_w^2 = 0.23$ are used in preparing this table. The precise definitions of the observables can be found in Refs. 7.

$$\begin{aligned}
R_\pi &\equiv \Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu) = R_\pi^{SM} (1 + 2\Delta_e - 2\Delta_\mu) \\
R_\tau &\equiv \Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\mu \rightarrow e\nu\bar{\nu}) = R_\tau^{SM} (1 + 2\Delta_\tau - 2\Delta_\mu) \\
R_{\mu\tau} &\equiv \Gamma(\tau \rightarrow \mu\nu\bar{\nu})/\Gamma(\mu \rightarrow e\nu\bar{\nu}) = R_{\mu\tau}^{SM} (1 + 2\Delta_\tau - 2\Delta_e) \\
\rho &= 1 + \alpha T \\
\sigma(\nu N \rightarrow \mu^- X) &= \sigma^{SM}(\nu N \rightarrow \mu^- X) \left[1 + 2\Delta_\mu - 2\Delta_e - 2 \operatorname{Re}(\delta\tilde{h}_R^{ud})/|V_{ud}| \right] \\
(g_L^2) &= (g_L^2)_{SM} - 0.00269 S + 0.00663 T - 1.452\Delta_\mu - 0.244\Delta_e \\
&\quad + 0.620 \operatorname{Re}(\delta\tilde{h}_R^{ud}) - 0.856 \delta\tilde{g}_L^{dd} + 0.689 \delta\tilde{g}_L^{uu} + 1.208 \delta\tilde{g}_L^{\nu\mu\nu\mu} \\
(g_R^2) &= (g_R^2)_{SM} + 0.000937 S - 0.000192 T + 0.085\Delta_e - 0.0359\Delta_\mu \\
&\quad + 0.0620 \operatorname{Re}(\delta\tilde{h}_R^{ud}) + 0.156 \delta\tilde{g}_R^{dd} - 0.311 \delta\tilde{g}_R^{uu} + 0.121 \delta\tilde{g}_L^{\nu\mu\nu\mu} \\
g_{eV} &= (g_{eV})_{SM} + 0.00723 S - 0.00541 T + 0.656\Delta_e + 0.730\Delta_\mu \\
&\quad + \delta\tilde{g}_L^{ee} + \delta\tilde{g}_R^{ee} - 0.074 \delta\tilde{g}_L^{\nu\mu\nu\mu} - 0.037 \operatorname{Re}(\delta\tilde{h}_R^{ud}) \\
g_{eA} &= (g_{eA})_{SM} - 0.00395 T + 1.012\Delta_\mu + \delta\tilde{g}_L^{ee} - \delta\tilde{g}_R^{ee} - 1.012 \delta\tilde{g}_L^{\nu\mu\nu\mu} - 0.0506 \operatorname{Re}(\delta\tilde{h}_R^{ud}) \\
C_{1u} &= C_{1u}^{SM} + 0.00482 S - 0.00493 T + 0.631(\Delta_e + \Delta_\mu) \\
&\quad + 0.387 \delta\tilde{g}_L^{ee} - \delta\tilde{g}_L^{uu} - 0.387 \delta\tilde{g}_R^{ee} - \delta\tilde{g}_R^{uu} \\
C_{1d} &= C_{1d}^{SM} - 0.00241 S + 0.00442 T - 0.565(\Delta_e + \Delta_\mu) \\
&\quad - 0.693 \delta\tilde{g}_L^{ee} - \delta\tilde{g}_L^{dd} + 0.693 \delta\tilde{g}_R^{ee} - \delta\tilde{g}_R^{dd} \\
C_{2u} &= C_{2u}^{SM} + 0.00723 S - 0.00544 T + 0.696(\Delta_e + \Delta_\mu) \\
&\quad + \delta\tilde{g}_L^{ee} - 0.08 \delta\tilde{g}_L^{uu} + \delta\tilde{g}_R^{ee} + 0.08 \delta\tilde{g}_R^{uu} \\
C_{2d} &= C_{2d}^{SM} - 0.00723 S + 0.00544 T - 0.696(\Delta_e + \Delta_\mu) \\
&\quad - \delta\tilde{g}_L^{ee} - 0.08 \delta\tilde{g}_L^{dd} - \delta\tilde{g}_R^{ee} + 0.08 \delta\tilde{g}_R^{dd} \\
Q_W(^{133}\text{Cs}) &= [Q_W(^{133}\text{Cs})]_{SM} - 0.796 S - 0.0113 T + 1.45(\Delta_e + \Delta_\mu) + 147 (\delta\tilde{g}_L^{ee} - \delta\tilde{g}_R^{ee}) \\
&\quad + 422 (\delta\tilde{g}_L^{dd} + \delta\tilde{g}_R^{dd}) + 376 (\delta\tilde{g}_L^{uu} + \delta\tilde{g}_R^{uu})
\end{aligned}$$

Table II: Low-Energy Observables

The contributions to low-energy ($q^2 \approx 0$) electroweak observables that are generated by the various interactions of the general non-oblique effective lagrangian. The precise definitions of the observables can be found in Ref. 8.

$$\begin{aligned}
m_W^2 &= (m_W^2)_{SM} [1 - 0.00723 S + 0.0111 T + 0.00849 U - 0.426(\Delta_e + \Delta_\mu)] \\
\Gamma_{\ell^+\ell^-} &= (\Gamma_{\ell^+\ell^-})_{SM} \left[1 - 0.00230 S + 0.00944 T - 1.209(\Delta_e + \Delta_\mu) \right. \\
&\quad \left. - 4.29 \delta\tilde{g}_L^{\ell\ell} + 3.66 \delta\tilde{g}_R^{\ell\ell} \right] \\
\Gamma_{u\bar{u}} &= (\Gamma_{u\bar{u}})_{SM} \left[1 - 0.00649 S + 0.0124 T - 1.59(\Delta_e + \Delta_\mu) \right. \\
&\quad \left. + 4.82 \delta\tilde{g}_L^{uu} - 2.13 \delta\tilde{g}_R^{uu} \right] \\
\Gamma_{d\bar{d}} &= (\Gamma_{d\bar{d}})_{SM} \left[1 - 0.00452 S + 0.0110 T - 1.41(\Delta_e + \Delta_\mu) \right. \\
&\quad \left. - 4.57 \delta\tilde{g}_L^{dd} + 0.828 \delta\tilde{g}_R^{dd} \right] \\
\Gamma_{b\bar{b}} &= (\Gamma_{b\bar{b}})_{SM} \left[1 - 0.00452 S + 0.0110 T - 1.41(\Delta_e + \Delta_\mu) \right. \\
&\quad \left. - 4.57 \delta\tilde{g}_L^{bb} + 0.828 \delta\tilde{g}_R^{bb} \right] \\
\Gamma_{\text{had}} &= (\Gamma_{\text{had}})_{SM} \left[1 - 0.00518 S + 0.0114 T - 1.469(\Delta_e + \Delta_\mu) \right. \\
&\quad - 1.01 (\delta\tilde{g}_L^{dd} + \delta\tilde{g}_L^{ss} + \delta\tilde{g}_L^{bb}) + 0.183 (\delta\tilde{g}_R^{dd} + \delta\tilde{g}_R^{ss} + \delta\tilde{g}_R^{bb}) \\
&\quad \left. + 0.822 (\delta\tilde{g}_L^{uu} + \delta\tilde{g}_L^{cc}) - 0.363 (\delta\tilde{g}_R^{uu} + \delta\tilde{g}_R^{cc}) \right] \\
\Gamma_{\nu_i\bar{\nu}_i} &= (\Gamma_{\nu_i\bar{\nu}_i})_{SM} \left[1 + 0.00781 T - (\Delta_e + \Delta_\mu) + 4 \delta\tilde{g}_L^{\nu_i\nu_i} \right] \\
\Gamma_Z &= (\Gamma_Z)_{SM} \left[1 - 0.00385 S + 0.0105 T - 1.35(\Delta_e + \Delta_\mu) + 0.574 (\delta\tilde{g}_L^{uu} + \delta\tilde{g}_L^{cc}) \right. \\
&\quad - 0.254 (\delta\tilde{g}_R^{uu} + \delta\tilde{g}_R^{cc}) + 0.268 (\delta\tilde{g}_L^{\nu_e\nu_e} + \delta\tilde{g}_L^{\nu_\mu\nu_\mu} + \delta\tilde{g}_L^{\nu_\tau\nu_\tau}) \\
&\quad - 0.144 (\delta\tilde{g}_L^{ee} + \delta\tilde{g}_L^{\mu\mu} + \delta\tilde{g}_L^{\tau\tau}) + 0.123 (\delta\tilde{g}_R^{ee} + \delta\tilde{g}_R^{\mu\mu} + \delta\tilde{g}_R^{\tau\tau}) \\
&\quad \left. - 0.707 (\delta\tilde{g}_L^{dd} + \delta\tilde{g}_L^{ss} + \delta\tilde{g}_L^{bb}) + 0.128 (\delta\tilde{g}_R^{dd} + \delta\tilde{g}_R^{ss} + \delta\tilde{g}_R^{bb}) \right] \\
A_{LR} &= A_{LR}^{SM} - 0.0284 S + 0.0201 T - 2.574(\Delta_e + \Delta_\mu) - 3.61 \delta\tilde{g}_L^{ee} - 4.238 \delta\tilde{g}_R^{ee} \\
A_{FB}^{\ell^+\ell^-} &= (A_{FB}^{\ell^+\ell^-})_{SM} - 0.00677 S + 0.00480 T - 0.614(\Delta_e + \Delta_\mu) \\
&\quad - 0.430 (\delta\tilde{g}_L^{ee} + \delta\tilde{g}_L^{\ell\ell}) - 0.505 (\delta\tilde{g}_R^{ee} + \delta\tilde{g}_R^{\ell\ell}) \\
A_{FB}(b\bar{b}) &= (A_{FB}(b\bar{b}))_{SM} - 0.0188 S + 0.0133 T - 1.70(\Delta_e + \Delta_\mu) \\
&\quad - 2.36 \delta\tilde{g}_L^{ee} - 2.77 \delta\tilde{g}_R^{ee} - 0.0322 \delta\tilde{g}_L^{bb} - 0.178 \delta\tilde{g}_R^{bb} \\
A_{FB}(c\bar{c}) &= (A_{FB}(c\bar{c}))_{SM} - 0.0147 S + 0.0104 T - 1.333(\Delta_e + \Delta_\mu) \\
&\quad - 1.69 \delta\tilde{g}_L^{ee} - 1.99 \delta\tilde{g}_R^{ee} + 0.175 \delta\tilde{g}_L^{cc} + 0.396 \delta\tilde{g}_R^{cc}
\end{aligned}$$

Table III: Weak-Scale Observables

The contributions to weak-scale ($q^2 = m_Z^2$ or m_W^2) electroweak observables that are generated by the various interactions of the general non-oblique effective lagrangian. The precise definitions of the observables can be found in Ref. 8.

Parameter	Individual Fit	Global Fit
S	-0.10 ± 0.16	-0.2 ± 1.0
T	$+0.01 \pm 0.17$	-0.02 ± 0.89
U	-0.14 ± 0.63	$+0.3 \pm 1.2$

Table IV: Oblique Parameters

Results for the oblique parameters S , T and U obtained from the fit of the new-physics parameters to the data. The second column gives the result for the (unrealistic) case where all other parameters are constrained to vanish. Column three gives the result of a global fit in which all of the parameters of the effective lagrangian are varied.

Parameter	Individual Fit	Global Fit
Δ_e	$-0.0008 \pm .0010$	$-0.0011 \pm .0041$
Δ_μ	$+0.00047 \pm .00056$	$+0.0005 \pm .0039$
Δ_τ	-0.018 ± 0.008	$-0.018 \pm .009$
$\text{Re}(\delta\tilde{h}_L^{ud})$	$-0.00041 \pm .00072$	$+0.0001 \pm .0060$
$\text{Re}(\delta\tilde{h}_R^{ud})$	$-0.00055 \pm .00066$	$+0.0003 \pm .0073$
$\text{Im}(\delta\tilde{h}_R^{ud})$	0 ± 0.0036	$-0.0036 \pm .0080$
$\text{Re}(\delta\tilde{h}_L^{us})$	$-0.0018 \pm .0032$	—
$\text{Re}(\delta\tilde{h}_R^{us})$	$-0.00088 \pm .00079$	$+0.0007 \pm .0016$
$\text{Im}(\delta\tilde{h}_R^{us})$	0 ± 0.0008	$-0.0004 \pm .0016$
$\text{Re}(\delta\tilde{h}_L^{ub}), \text{Im}(\delta\tilde{h}_L^{ub})$	$-0.09 \pm .16$	—
Σ_1	—	$+0.005 \pm .027$
$\text{Re}(\delta\tilde{h}_R^{ub})$	—	—
$\text{Re}(\delta\tilde{h}_L^{cd})$	$+0.11 \pm .98$	—
$\text{Re}(\delta\tilde{h}_R^{cd})$	—	—
$\text{Re}(\delta\tilde{h}_L^{cs})$	$+0.022 \pm .20$	—
$\text{Re}(\delta\tilde{h}_R^{cs})$	$+0.022 \pm .20$	—
$\text{Re}(\delta\tilde{h}_L^{cb})$	$+0.5 \pm 4.6$	—
Σ_2	—	$+0.11 \pm 0.98$
$\text{Re}(\delta\tilde{h}_R^{cb})$	—	—

Table V: Charged-Current Parameters

More results of the fits of the new-physics parameters to the data. The quantities Σ_1 and Σ_2 arise in tests for the unitarity of the CKM matrix, and are defined as: $\Sigma_1 \equiv \text{Re}(\delta\tilde{h}_L^{us}) + [\text{Re}(V_{ub})\text{Re}(\delta\tilde{h}_L^{ub}) + \text{Im}(V_{ub})\text{Im}(\delta\tilde{h}_L^{ub})] / |V_{us}|$ and $\Sigma_2 \equiv \text{Re}(\delta\tilde{h}_L^{cd}) + |V_{cs}|\text{Re}(\delta\tilde{h}_L^{cs} + \delta\tilde{h}_R^{cs}) / |V_{cd}| + |V_{cb}|\text{Re}(\delta\tilde{h}_L^{cb}) / |V_{cd}|$. Blanks indicate where the corresponding fit would be inappropriate, such as for when a parameter always appears in a particular combination with others, and so cannot be individually fit.

Parameter	Individual Fit	Global Fit
$\delta\tilde{g}_L^{dd}$	$+0.0016 \pm .0015$	$+0.003 \pm .012$
$\delta\tilde{g}_R^{dd}$	$+0.0037 \pm .0038$	$+0.007 \pm .015$
$\delta\tilde{g}_L^{uu}$	$-0.0003 \pm .0018$	-0.002 ± 0.014
$\delta\tilde{g}_R^{uu}$	$+0.0032 \pm .0032$	$-0.003 \pm .010$
$\delta\tilde{g}_L^{ss}$	$-0.0009 \pm .0017$	$-0.003 \pm .015$
$\delta\tilde{g}_R^{ss}$	$-0.0052 \pm .00095$	$+0.002 \pm .085$
$\delta\tilde{g}_L^{cc}$	$-0.0011 \pm .0021$	$+0.001 \pm .018$
$\delta\tilde{g}_R^{cc}$	$+0.0028 \pm .0047$	$+0.009 \pm .029$
$\delta\tilde{g}_L^{bb}$	$-0.0005 \pm .0016$	$-0.0015 \pm .0094$
$\delta\tilde{g}_R^{bb}$	$+0.0019 \pm .0083$	$0.013 \pm .054$
$\delta\tilde{g}_L^{\nu_e\nu_e}$	$-0.0048 \pm .0052$	—
$\delta\tilde{g}_L^{\nu_\mu\nu_\mu}$	$-0.0021 \pm .0027$	$+0.0023 \pm .0097$
$\delta\tilde{g}_L^{\nu_\tau\nu_\tau}$	$-0.0048 \pm .0052$	—
$\delta\tilde{g}_L^{\nu_e\nu_e} + \delta\tilde{g}_L^{\nu_\tau\nu_\tau}$	—	$-0.004 \pm .033$
$\delta\tilde{g}_L^{ee}$	$-0.00029 \pm .00043$	$-0.0001 \pm .0032$
$\delta\tilde{g}_R^{ee}$	$-0.00014 \pm .00050$	$+0.0001 \pm .0030$
$\delta\tilde{g}_L^{\mu\mu}$	$+0.0040 \pm .0051$	$+0.005 \pm .032$
$\delta\tilde{g}_R^{\mu\mu}$	$-0.0003 \pm .0047$	$+0.001 \pm .028$
$\delta\tilde{g}_L^{\tau\tau}$	$-0.0021 \pm .0032$	$0.000 \pm .022$
$\delta\tilde{g}_R^{\tau\tau}$	$-0.0034 \pm .0028$	$-0.0015 \pm .019$

Table VI: Neutral-Current Parameters

Still more results of the fits of the new-physics parameters to the data. As before, blanks indicate where the corresponding fit would be inappropriate, such as for when a parameter always appears in a particular combination with others, and so cannot be individually fit.

Parameter	One-Loop Result
S	$2.63\Delta g_{1Z} - 2.98\Delta\kappa_\gamma + 2.38\Delta\kappa_Z + 5.97\lambda_\gamma - 4.50\lambda_Z$
T	$-1.82\Delta g_{1Z} + 0.550\Delta\kappa_\gamma + 5.83\Delta\kappa_Z$
U	$2.42\Delta g_{1Z} - 0.908\Delta\kappa_\gamma - 1.91\Delta\kappa_Z + 2.04\lambda_\gamma - 2.04\lambda_Z$
V	$0.183\Delta\kappa_Z$
W	$0.202\Delta g_{1Z}$
X	$-0.0213\Delta\kappa_\gamma - 0.0611\Delta\kappa_Z$
\hat{V}	$0.183\Delta\kappa_Z - (3.68\Delta g_{1Z} + 0.797\Delta\kappa_Z)(m_t^2/m_W^2)$
\hat{X}	$-0.0213\Delta\kappa_\gamma - 0.0611\Delta\kappa_Z + (0.423\Delta g_{1Z} + 0.0916\Delta\kappa_Z)(m_t^2/m_W^2)$

TABLE VII: TGV Contributions to Oblique Parameters

One-loop results for the induced parameters S , T , U , V , W and X , defined at $\mu = 100$ GeV, in terms of the various TGV couplings defined at $M = 1$ TeV. As usual, $\alpha(m_Z^2) = 1/128$ and $s_w^2 = 0.23$.

Figure Captions

- Figure 1:** Constraints on S and T from a fit to both high- and low-energy electroweak measurements. The solid line represents the 68% C.L. setting VWX to zero, the dashed line represents the 90% C.L. setting VWX to zero, the dotted line represents the 68% C.L. allowing VWX to vary, and the dot-dashed line represents the 90% C.L. allowing VWX to vary.
- Figure 2:** Constraints on S' and T' from a global fit of precision electroweak measurements at the Z resonance only. The two lines represent the 68% C.L. and 90% C.L. ellipses.
- Figure 3:** The contribution of an extra SM family to the oblique parameters S' and T' . Solid line: a colour-triplet, $Y = \frac{1}{6}$ quark doublet with masses $(m_{b'}, m_{t'})$ as indicated in brackets. Dotted line: a colour-singlet, $Y = \frac{1}{2}$ lepton doublet with masses $(m_{\nu'}, m_{l'})$. The grid spacing represents steps of 25 (resp. 30) GeV for the solid (resp. dotted) plots.

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